

# Functioning of Dielectric Dimensions in Gaussian Mixture Framework for Biomedical Tissue

Jayeeta Ukil

*Abstract- The functioning of dielectric dimensions in Finite-difference time-domain (FDTD) method used to design the cell model which calculates the magnetic field in finite position for an impulse response source placed in the point. This source is inserted in a medium characterized by dielectric permittivity  $\epsilon$ , magnetic permeability  $\mu$  and conductivity sigma. These parameters are having better results while we are adopting proposed Gaussian Mixture Model for Biomedical tissue.*

*Index Terms— Dielectric Dimensions, Biological Tissues, Cell Model, FDTD, GMM.*

## I. INTRODUCTION

Frequency changes in an electromagnetic wave in a time-varying plasma has been extensively studied [1]–[7] and some experiments [1]–[3] were carried out to demonstrate frequency shifting. Analytical studies make various assumptions and use simplified geometries including one-dimensional models, flash ionization, and slow or fast creation of the plasma medium [4]–[7]. A limited number of theoretical and numerical studies of three-dimensional (3-D) models are reported. Buchsbaum *et al.* [8] examined the perturbation theory for various mode configurations of a cylindrical cavity co axial with a plasma column and coaxial with the static magnetic field. Gupta used a moment method to study cavities and wave guides containing an isotropic media and compared the results with the perturbation method [9]. A transmission-line-matrix (TLM) method was developed to study the interaction of an electromagnetic wave with time-invariant and space-invariant magnetized plasma in 3-D space [10]. Mendonca [11] presented a mode coupling theory in a cavity for space-varying and slowly created isotropic plasma. Since the introduction of the finite-difference time-domain (FDTD) method [12], it has been widely used in solving many electromagnetic problems including those concerned with plasma media [13], [14]. For anisotropic cases [15], [16], the equations for the components of the current density vector become coupled and the implementation of the conventional FDTD scheme is difficult.

We propose a new FDTD method to overcome this difficulty. In this paper, we use the GMM method to analyze the interaction of an electromagnetic wave with a magneto plasma medium created in a cavity. The FDTD algorithm is derived first and the implementation of perfect electric conductor (PEC) boundary conditions is investigated as an

existing method. The Proposed GMM algorithm is illustrated by computing the new frequencies. Then in the comparison we have shown the better results of GMM in dielectric medium.

## II. FINITE-DIFFERENCE TIME-DOMAIN (FDTD)

Finite-difference time-domain (FDTD) is one of the primary available computational electrodynamics modeling techniques. Since it is a time-domain method, FDTD solutions can cover a wide frequency range with a single simulation run, and treat nonlinear material properties in a natural way. The FDTD method belongs in the general class of grid-based differential time-domain numerical modeling methods. The time-dependent Maxwell's equations (in partial differential form) are discretized using central-difference approximate to the space and time partial derivatives. The resulting finite-difference equations are solved in either software or hardware in a leapfrog manner: the electric field vector components in a volume of space are solved at a given instant time; then the magnetic field vector components in the same spatial volume are solved at the next instant in time; and this process is repeated over and over again until the desired transient or steady-state electromagnetic field behavior is fully evolved.

The basic FDTD space grid and time-stepping algorithm trace back to a seminal 1966 paper by Kane Yee in IEEE Transactions on Antennas and Propagation. The descriptor "Finite-difference time-domain" and its corresponding "FDTD" acronym were originated by Allen Taflov in a 1980 paper in IEEE Transactions on Electromagnetic Compatibility. When Maxwell's differential equations are examined, it is seen that the change in the E-field in time (the time derivative) is dependent on the change in the H-field across the space. This results in the basic FDTD time-stepping relation that, at any point in space, the updated value of the E-field in time is dependent on the stored value of the E-field and the numerical curl of the local distribution of the H-field in space.

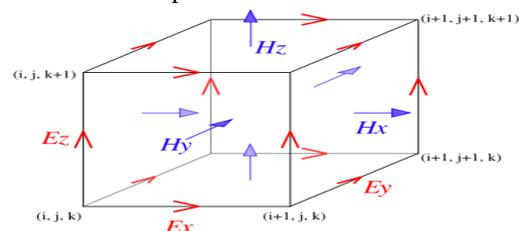


Fig1. H-field in space

The H-field is explained in a similar manner in fig1. At any point in space, the updated value of the H-field in time is dependent on the stored value of the H-field and the numerical curl of the local distribution of the E-field in space. Illustrating the E-field and H-field updates resulting in a marking-in-time process wherein sampled-data analogs of the continuous electromagnetic waves under consideration propagate in a numerical grid stored in the computer memory. This description holds true for 1-D, 2-D, and 3-D FDTD techniques. When multiple dimensions are considered, calculating the numerical curve can become complicated. Kane Yee's seminal 1966 paper proposed spatially staggering the vector components of the E-field and H-field about rectangular unit cells of a Cartesian computational grid so that each E-field vector component is located midway between a pair of H-field vector components, and conversely.<sup>[1]</sup> This scheme, now known as a Yee lattice, had proven to be very robust, and remains at the core of many current FDTD software constructs. Furthermore, Yee proposed a leapfrog scheme for marching in time wherein the E-field and H-field updates are staggered so that E-field updates are conducted midway during each time-step between successive H-field updates, and conversely. [17] On the positive side, this explicit time-stepping scheme avoids the need to solve simultaneous equations, and furthermore yields dissipation-free numerical wave propagation. On the negative side, this scheme mandates an upper bound on the time-step to ensure numerical stability.<sup>[4]</sup> As a result, certain classes of simulations can require many thousands of steps for completion

To implement an FDTD solution of Maxwell's equations, a computational domain must first be established. The computational domain is simply the physical region over which the simulation will be performed. The E and H fields are determined at every point in space within that computational domain. The material of each cell within the computational domain must be specified. Typically, the material is either free-space (air), metal, or dielectric. Any material can be used as long as the permeability, permittivity, and conductivity are specified. Once the computational domain and the grid materials are established, a source is specified. The source can be an impinging plane wave, a current on a wire, or an applied electric field, depending on the application. Since the E and H fields are determined directly, the output of the simulation is usually the E or H field at a point or a series of points within the computational domain. The simulation evolves the E and H fields forward in time. Processing may be done on the E and H fields returned by the simulation. Data processing may also occur while the simulation is ongoing. While the FDTD technique computes electromagnetic fields within a compact spatial region, scattered and/or radiated far fields can be obtained

via near-to-far-field transformations. Every technique has strengths and weaknesses, and the FDTD method is not different.

FDTD is a versatile modeling technique used to solve Maxwell's equations. It is intuitive, so users can easily understand how to use it and know what to expect from a given model. FDTD is a time-domain technique, and when a broadband pulse (such as a Gaussian pulse) is used as the source, and then the response of the system over a wide range of frequencies can be obtained with a single simulation. This is useful in applications where resonant frequencies are not exactly known, or anytime that a broadband result is desired. Since FDTD calculates the E and H fields everywhere in the computational domain as they evolve in time, it lends itself to providing animated displays of the electromagnetic field movement through the model. This type of display is useful in understanding what is going on in the model, and to help to ensure that the model is working correctly. The FDTD technique allows the user to specify the material at all points within the computational domain. A wide variety of linear and nonlinear dielectric and magnetic materials can be naturally and easily modeled. FDTD allows the effects of aperture to be determined directly. Shielding effects can be found, and the fields both inside and outside a structure can be found directly or indirectly. FDTD uses the E and H fields directly. Since most EMI/EMC modeling applications are interested in the E and H fields, it is convenient that no conversions must be made after the simulation has run to get these values. Since FDTD requires that the entire computational domain be gridded, and the grid spatial discretization must be sufficiently fine to resolve both the smallest electromagnetic wavelength and the smallest geometrical feature in the model, very large computational domains can be developed, which results in very long solution times. Models with long, thin features, (like wires) are difficult to model in FDTD because of the excessively large computational domain required.

There is no way to determine unique values for permittivity and permeability at a material interface. Space and time steps must satisfy the CFL condition. FDTD finds the E/H fields directly everywhere in the computational domain. If the field values at some distance are desired, it is likely that this distance will force the computational domain to be excessively large. Far-field extensions are available for FDTD, but require some amount of post processing.<sup>[3]</sup> Since FDTD simulations calculate the E and H fields at all points within the computational domain, the computational domain must be finite to permit its residence in the computer memory. In many cases this is achieved by inserting artificial boundaries into the simulation space. So, Care must be taken to minimize errors introduced by such boundaries. There are a number of available highly effective absorbing boundary

conditions (ABCs) to simulate an infinite unbounded computational domain.<sup>[3]</sup> Most modern FDTD implementations instead use a special absorbing "material", called a perfectly matched layer (PML) to implement absorbing boundaries.<sup>[6][7]</sup> Because FDTD is solved by propagating the fields forward in the time domain, the electromagnetic time response of the medium must be modeled exactly. For an arbitrary response, this involves a computationally expensive time convolution, although in most cases the time response of the medium (or Dispersion (optics)) can be adequately and simply modeled using either the recursive convolution (RC) technique, the auxiliary differential equation (ADE) technique, or the Z-transform technique. An alternative way of solving Maxwell's equations that can treat arbitrary dispersion easily is the Pseudo spectral Spatial-Domain method (PSSD), which instead propagates the fields forward in space.

### III. IMPLEMENTATION OF GAUSSIAN MIXTURE MODEL

In statistics, a mixture model for representing the presence of sub-populations within an overall population, without requiring that an observed data-set should identify the sub-population to which an individual observation belongs. Formally a mixture model corresponds to the mixture distribution that represents the probability distribution of observations in the overall population. However, while problems associated with "mixture distributions" relate to deriving the properties of the overall population from those of the sub-populations, "mixture models" are used to make statistical inferences about the properties of the sub-populations given only observations on the pooled population, without sub-population-identified information.

Some ways of implementing mixture models involve steps that attribute to the postulated sub-population-identities to individual observations (or weights towards such sub-populations), in case these can be regarded as types of unsupervised learning or clustering procedures. However not all inference procedures involve such steps.

A typical finite-dimensional mixture model is a hierarchical model consisting of the following components:

- $N$  random variables corresponding to observations, each assumed to be distributed according to a mixture of  $K$  components, with each component belonging to the same parametric family of distributions but with different parameters
- $N$  corresponding random latent variables specifying to identify the mixture component of each observation, each distributed according to a  $K$ -dimensional categorical distribution
- A set of  $K$  mixture weights, each of which is a probability (a real number between 0 and 1), all of which sum to 1

- A set of  $K$  parameters, each specifying the parameter of the corresponding mixture component. In many cases, each "parameter" is actually a set of parameters.

For example, observations distributed according to a mixture of one-dimensional Gaussian distributions will have a mean and variance for each component. Observations distributed according to a mixture of  $V$ -dimensional categorical distributions (e.g., when each observation is a word from a vocabulary of size  $V$ ) will have a vector of  $V$  probabilities, collectively summing to 1. In addition, in a Bayesian setting, the mixture weights and parameters will themselves be random variables, and prior distributions will be placed over the variables. In such a case, the weights are typically viewed as a  $K$ -dimensional random vector drawn from a Dirichlet distribution (the conjugate prior of the categorical distribution), and the parameters will be distributed according to their respective conjugate priors.

A typical non-Bayesian Gaussian mixture model looks like this:

$$\begin{aligned}
 K, N &= \text{as above} \\
 \phi_{i=1\dots K}, \phi &= \text{as above} \\
 z_{i=1\dots N}, x_{i=1\dots N} &= \text{as above} \\
 \mu_{i=1\dots K} &= \text{mean of component } i \\
 \sigma_{i=1\dots K}^2 &= \text{variance of component } i \\
 z_{i=1\dots N} &\sim \text{Categorical}(\phi) \\
 x_{i=1\dots N} &\sim \mathcal{N}(\mu_{z_i}, \sigma_{z_i}^2)
 \end{aligned}$$

A Bayesian version of a Gaussian mixture model is as follows:

$$\begin{aligned}
 K, N &= \text{as above} \\
 \phi_{i=1\dots K}, \phi &= \text{as above} \\
 z_{i=1\dots N}, x_{i=1\dots N} &= \text{as above} \\
 \mu_{i=1\dots K} &= \text{mean of component } i \\
 \sigma_{i=1\dots K}^2 &= \text{variance of component } i \\
 \mu_0, \lambda, \nu, \sigma_0^2 &= \text{shared hyperparameters} \\
 \mu_{i=1\dots K} &\sim \mathcal{N}(\mu_0, \lambda\sigma_i^2) \\
 \sigma_{i=1\dots K}^2 &\sim \text{Inverse-Gamma}(\nu, \sigma_0^2) \\
 \phi &\sim \text{Symmetric-Dirichlet}_K(\beta) \\
 z_{i=1\dots N} &\sim \text{Categorical}(\phi) \\
 x_{i=1\dots N} &\sim \mathcal{N}(\mu_{z_i}, \sigma_{z_i}^2)
 \end{aligned}$$

A typical non-Bayesian mixture model with categorical observation looks like this:

- $K, N$  : as above
- $\phi_{i=1\dots K}, \phi$  : as above
- $z_{i=1\dots N}, x_{i=1\dots N}$  : as above
- $V$  : Dimension of categorical observations, e.g., size of word vocabulary
- $\theta_{i=1\dots K, j=1\dots V}$  : probability for component  $i$  of observing item  $j$

- $\theta_{i=1...K}$  :vector of dimension  $V$ , composed of  $\theta_{i,1...V}$  must sum to 1

The random variables:

$$z_{i=1...N} \sim \text{Categorical}(\phi)$$

$$x_{i=1...N} \sim \text{Categorical}(\theta_{z_i})$$

A typical Bayesian mixture model with categorical observations looks like this:

- $K, N$  :as above
- $\phi_{i=1...K}, \phi$  :as above
- $z_{i=1...N}, x_{i=1...N}$  :as above
- $V$  :dimension of categorical observations, e.g., size of word vocabulary
- $\theta_{i=1...K, j=1...V}$  :probability for component  $i$  of observing item  $j$
- $\theta_{i=1...K}$  :vector of dimension  $V$ , composed of  $\theta_{i,1...V}$  must sum to 1
- $\alpha$  :shared concentration hyper parameter of  $\theta$  for each component
- $\beta$  :concentration hyper parameter of  $\phi$

The random variables:

$$\phi \sim \text{Symmetric-Dirichlet}_K(\beta)$$

$$\theta_{i=1...K} \sim \text{Symmetric-Dirichlet}_V(\alpha)$$

$$z_{i=1...N} \sim \text{Categorical}(\phi)$$

$$x_{i=1...N} \sim \text{Categorical}(\theta_{z_i})$$

#### IV. SIMULATION RESULTS

##### FDTD Simulation Results

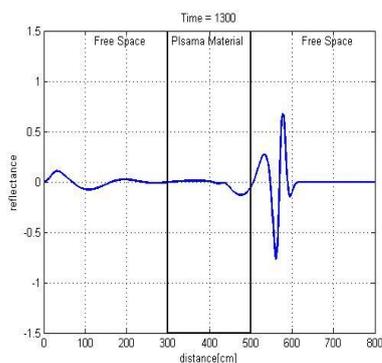
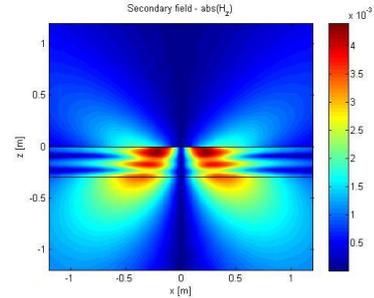
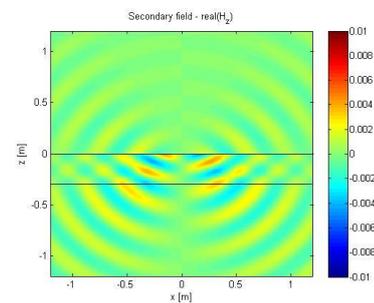


Fig2. This figure represents how signal behaves in the plasma material

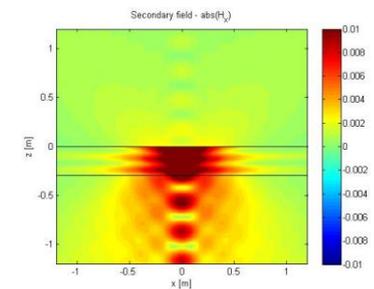
##### Biomedical Tissue Simulation Results



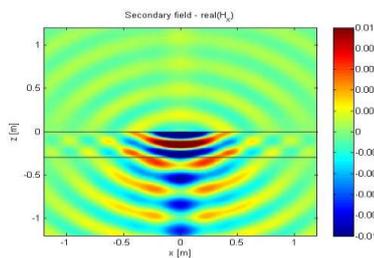
a) Frame 1



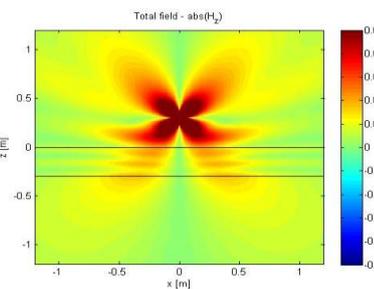
b) Frame 2



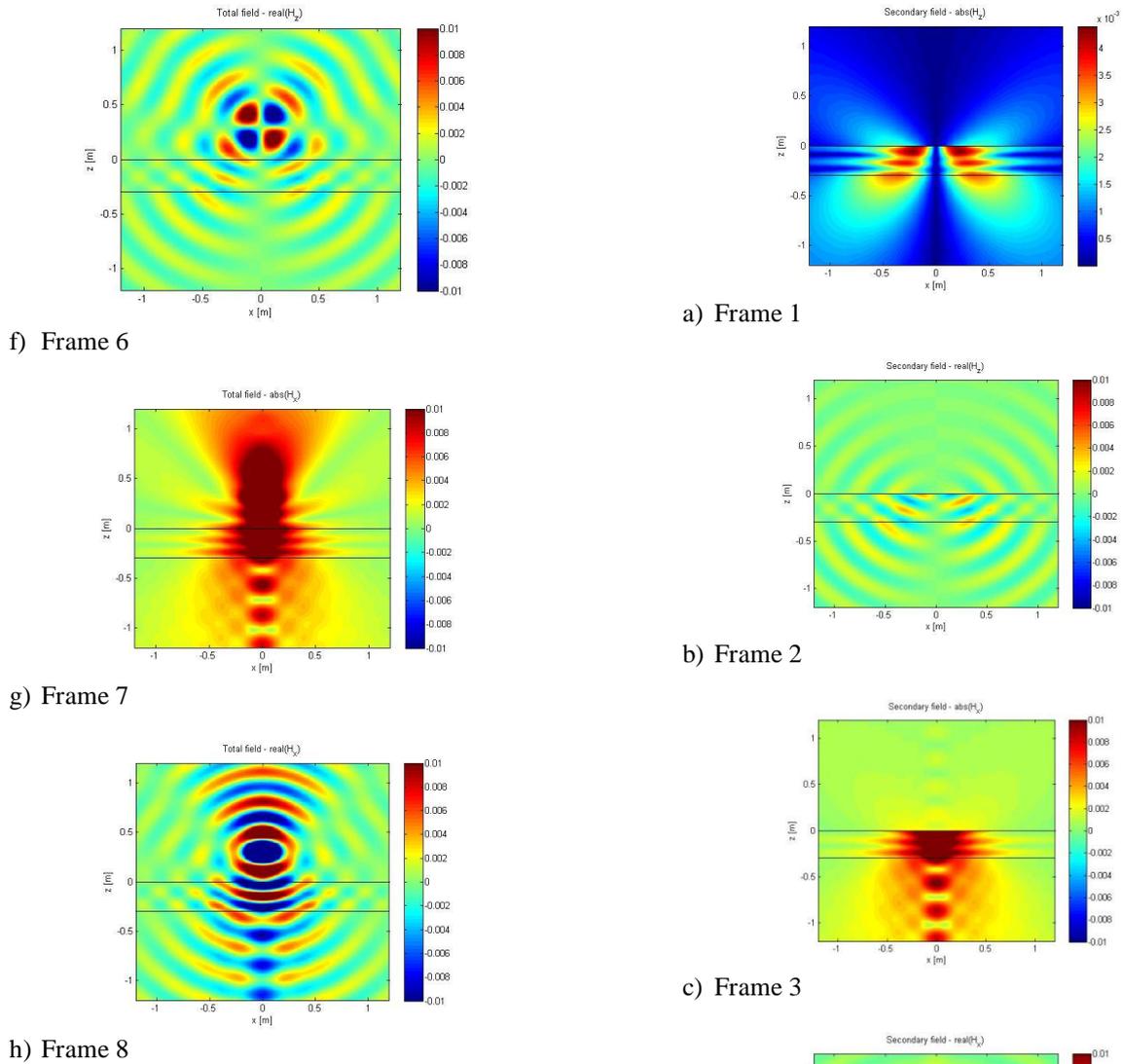
c) Frame 3



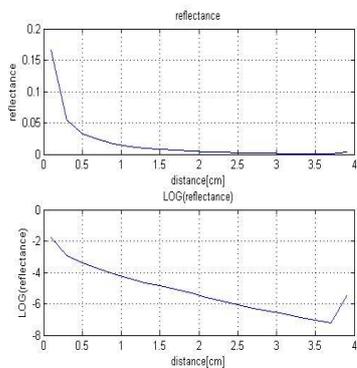
d) Frame 4



e) Frame 5



**Fig 3.** This figure represents the behaviour of secondary field in dielectric medium of tissue in different Frames from a) to h) GMM Results



**Fig4.** This figure represents the performance of reflectance in the dielectric medium after applying Gaussian Mixture Model

**Fig 5.** This figure represents the behaviour of secondary field in dielectric medium of tissue with GMM in different Frames from a) to d)

### V. CONCLUSION

The GMM performance in dielectric medium is better than FDTD method where in FDTD we cannot process secondary medium which is possible in GMM method. When we divide dielectric in to frames the formation secondary is most important to calculate the  $\epsilon$  so it's possible only in GMM method.

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